Reform proposals for occupational plans and state pension schemes

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1 Introduction

Time is not just Money:

There are no pockets in a shroud.

Pensions

A **pension plan** is a financial contract between a pension provider and the member(s) of the plan, established for the purpose of providing an income in retirement for the member(s).

- State pension plans (social security plans). Membership is usually compulsory for people who work in the country (usually pay-as-you-go $\leftrightsquigarrow$ PAYG).
  $\Rightarrow$ **First Pillar**
- Group pension plans: cover a number of individuals who share a common interest (working for the same employer who sets up the plan on their behalf). Occupational pension, often funding based.
  $\Rightarrow$ **Second Pillar**
- Single-member plan: insurance contracts taken out by an individual for the purpose of saving for retirement. Individual savings, personal plan.
  $\Rightarrow$ **Third Pillar**

Classification of Pension Schemes...

The fundamental choice (pay-as-you-go/funding) is present for both: defined benefit (DB) and defined contributions pension schemes (DC).

	Pay-As-You-Go	**Funding**
DB	Classical social security	Classical employee benefit DB plan
DC	Notional accounts (NDCs)	Pension saving accounts

Risks

General Risks

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COVID-19

The COVID-19 is not just a public health issue.
It is having a major long-term impact on the economy and the financial system.

For example, the German Pension Insurance expects in 2020 a loss of approximately **4.7 billion euro**.

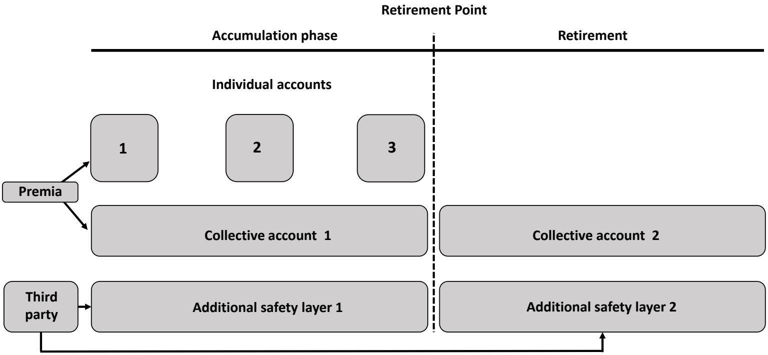
Photo by Bernhard Stärck, https://pixabay.com

A popular question in Germany:

To live now or after the retirement?

An Alternative Scheme

(Boado-Penas, E., Helmert & Krühner)



The overall target of the maximal with-profit is twofold:

• to maximise the total saved amount (alternatively the first pension or the discounted pension payments expected at the retirement point)

• to keep the pension evolution inside a corridor, mitigating the risk of a decrease.

Unit-linked Contracts

With a unit-linked policy, the premia buy units in the fund of the investor's choice.
This might be run by the life office itself, or it might be a unit trust or open ended investment company run by the life office or another institution.

It has the following characteristics:

- The value of the policy is measured by the total value of the units allocated to it.
- A policy's value depends on the performance of the fund, or funds, to which it is linked.

Annuity Pools

An annuity is a contract between an annuitant and an insurance company, who promises to pay a certain amount of money on a periodic basis.

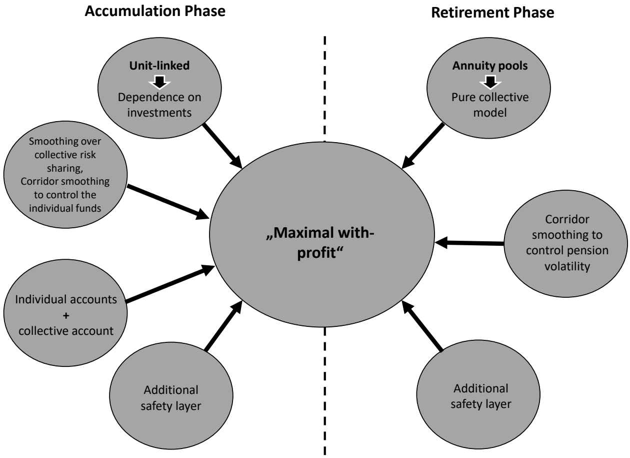
- The annuity provides a retirement-income insurance: contributions to the annuity in exchange for an income stream later.
- The goal of an annuity is to provide a stable, long-term income supplement for the annuitant.
- There is a redistribution of assets / reserve from those who die before average to those who live longer.
- If life expectancy of the customers increases on the average more as calculated, pensions / benefits decrease.

Degree of Capital Cover (DCC)

In the retirement phase, we will control the evolution of the pensions via the degree of capital coverage (DCC) of the collective fund.
For the calculation of DCC, we have to consider the time value of the collective fund and the present value of pensions to be paid. The relation between both is a good measure to adjust the annuities and gives us:

$$\text{DCC} = \frac{\text{The value of the collective fund}}{\text{Pensions to be paid}} .$$

The corridor we are looking for is an interval surrounding the DCC. An example is given by the German law (BRSG): $100\% \leq$DCC$\leq 125\%$.
Thus, if DCC exits the corridor, the pensions have to be adjusted.
However, the third layer can prevent or at least postpone the downward adjustments.



Accumulation Phase
Retirement Phase
Unit-linked
Dependence on investments
Annuity pools
Pure collective model
Smoothing over collective risk sharing, Corridor smoothing to control the individual funds
„Maximal with-profit“
Corridor smoothing to control pension volatility
Individual accounts + collective account
Additional safety layer
Additional safety layer

Corridor Smoothing of the Fund H

Let $V(t)$ denote the value of an individual account and $\eta(t)$ the number of shares at time t.

- In case of an **overperformance**, i.e. $\frac{H_t}{H_{t-1}} - 1 > k$, one has to transfer into collective account

$$\frac{1}{4}\Big(H_t - H_{t-1}(1+k)\Big)\eta(t-1) = \frac{1}{4}V(t-1)\Big(\frac{H_t}{H_{t-1}} - 1 - k\Big),$$

- In case of an **underperformance**, i.e. $\frac{H_t}{H_{t-1}} - 1 < -k$, the individual account creates a claim of

$$\frac{1}{2}\Big(H_{t-1}(1-k) - H_t\Big)\eta(t-1) = \frac{1}{2}V(t-1)\Big(1 - k - \frac{H_t}{H_{t-1}}\Big)\ ,$$

Profitability condition:

The set of admissible $k \in [0,1]$ is given by those k such that

$$\mathbb{E}\left[\frac{1}{2}\left(1-k-\frac{H_t}{H_{t-1}}\right)^{+}-\frac{1}{4}\left(\frac{H_t}{H_{t-1}}-1-k\right)^{+}\right] \leq 0\,.$$

The reason for restricting the set of the admissible k is that the collective fund should not ruin almost surely due to the withdrawals from the individual funds.

This condition can be compared to the net profit condition in ruin theory.

Redistribution Index

It is important to give a cohort that leaves the saving phase a fair share of the accumulated collective wealth = Collective account 1.

A possible solution would be to put weights on account values in different time intervals: the earlier a value the higher the weight.

We denote this redistribution index by $J_i(t)$ where i is the number of the insurance contract.

The Individual and Collective Accounts

The value of the individual account i is given as follows

$$\boldsymbol{V_i(t)} = \gamma\pi + \eta_i(t-1)H_t - \frac{1}{4}V_i(t-1)\left(\frac{H_t}{H_{t-1}} - 1 - k\right)^+ + \frac{1}{2}V_i(t-1)\left(1 - k - \frac{H_t}{H_{t-1}}\right)^+ .$$

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The collective account is then described by

$$\boldsymbol{C(t)} = (1-\gamma)\Pi + \theta(t-1)H_t + \frac{1}{4}\sum_{j=1}^{n} V_j(t-1)\left(\frac{H_t}{H_{t-1}} - 1 - k_j\right)^+ - \frac{1}{2}\sum_{j=1}^{n} V_j(t-1)\left(1 - k_j - \frac{H_t}{H_{t-1}}\right)^+ .$$

Maximising the Expected Total Capital at Retirement

$$\mathbb{E}[V(t) + J(t-1)C(t)] \to \max!$$

Considering just the part depending on k, we have to maximise the function

$$\mathbb{E}\left[\frac{1}{2}\left(1 - k - \frac{H_T}{H_{T-1}}\right)^+ - \frac{1}{4}\left(\frac{H_T}{H_{T-1}} - 1 - k\right)^+\right].$$

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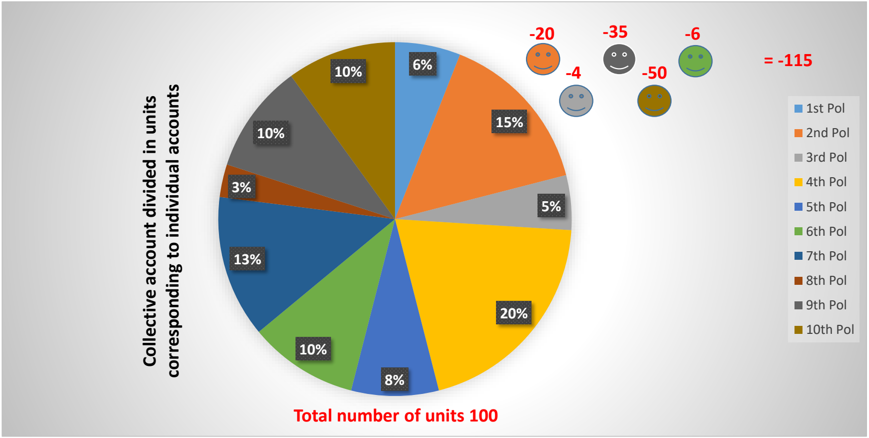
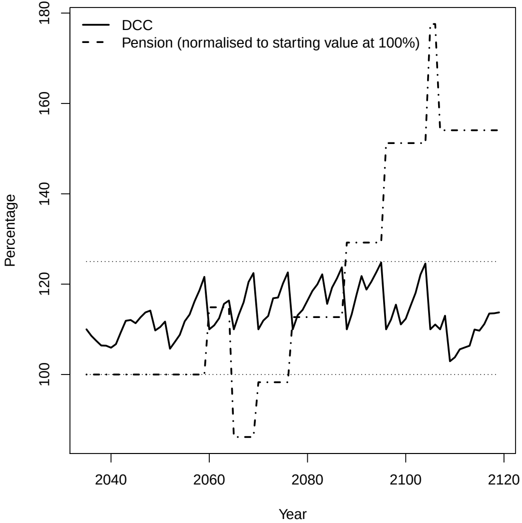
One may want to **penalise the variance**:The redistribution index becomes important!

Different Strategies For Getting Help From the Collective Account



A Possible Pension Evolution



Imagine...

Aim: A mixed scheme with a credit granted by the state

(Boado-Penas, E. & Korn)

- We aim to propose a mixed pension system that consists of a combination of a classical pay-as-you-go (PAYG) scheme and an extra amount of contributions invested in a funding scheme.
- The investment of funding part is designed so that the PAYG system is financially sustainable at a particular level of probability and at the same time provide some gains to individuals.

The Model

- We consider a **prototypical contributor (PC)** , i.e. *the average contributor*, with an average salary and average salary increases.

- This PC has to contribute an amount C_0 – expressed in percentage of his salary – at time $t = 0$ into the PAYG.

- Assume that the state anticipates a deficit and it is known that the contributions that make the PAYG system sustainable in the future until some deterministic time T are given by

$$C = (C_1, ..., C_T)$$

where

$$C_j \geq C_0, \; j = 1, ..., T \; .$$

Sustainability via a Credit

The classical PAYG scheme is transformed in the following way:

- In the next T years the PC pays C_0 into the PAYG. The state is taking over the payment of the differences $C_j - C_0$ for $j = 1, ..., T$.

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Lump Sum Repayment

The contribution to PAYG of the PC is fixed to be C_0 for the next T years. The PC invests an amount of $\alpha(C_j - C_0)$, $\alpha > 0$, into a **fund** with dynamics

$$F_t = F_0 e^{\mu t + \sigma W_t}$$

where $\mu, \sigma > 0$ and W is a standard Brownian motion.

At time $j+1$:

The PC pays $C_j - C_0$ to the state if

$$\alpha(C_j - C_0)e^{\mu + \sigma(W_{j+1} - W_j)} \geq C_j - C_0 \; .$$

If this is the case, then the remaining value of the fund position stays with the PC.

Otherwise, the state receives the full fund position and takes over the loss of

$$L_j := (C_j - C_0) \cdot \left(1 - \alpha e^{\mu + \sigma(W_{j+1} - W_j)}\right) \; .$$

Lump Sum Repayment: Payback first

For a value of $\alpha > 0$ and $C_j > C_0 > 0$, we obtain:

a) The probability that the full payment is made to the state at time j is given by

$$\mathbb{P}\left[\alpha e^{\mu+\sigma(W_{j+1}-W_j)} \geq 1\right] = \Phi\left(\frac{\mu + \ln(\alpha)}{\sigma}\right) .$$

b) The expected loss $\mathbb{E}[L_j]$ of the state at time j is given by

$$\begin{aligned}\mathbb{E}[L_j] &= \mathbb{E}\left[(C_j - C_0)\left(1 - \alpha e^{\mu+\sigma(W_{j+1}-W_j)}\right)^+\right] \\ &= (C_j - C_0)\left\{\Phi\left(-\frac{\mu + \ln(\alpha)}{\sigma}\right) - \alpha e^{\mu+\frac{\sigma^2}{2}}\Phi\left(-\frac{\mu + \sigma^2 + \ln(\alpha)}{\sigma}\right)\right\} .\end{aligned}$$

c) The expected gain $\mathbb{E}[G_j]$ of the PC at time j is given by

$$\begin{aligned}\mathbb{E}[G_j] &= \mathbb{E}\left[(C_j - C_0)\left(\alpha e^{\mu+\sigma(W_{j+1}-W_j)} - 1\right)^+\right] \\ &= (C_j - C_0)\left\{\alpha e^{\mu+\frac{\sigma^2}{2}}\Phi\left(\frac{\mu + \sigma^2 + \ln(\alpha)}{\sigma}\right) - \Phi\left(\frac{\mu + \ln(\alpha)}{\sigma}\right)\right\} .\end{aligned}$$

Continuous Repayment: The Repayment Procedure

Here, we assume the same credit scheme, but a different set of repayment strategies:

- We assume that the state requires the PC to share the profits from an investment continuously in time, i.e. the PC has to transfer any excess above some level b into a special bank account during a 1-year period.
- The state pays the difference between the old and the new contributions in the following T years.
  The PC has to invest a certain amount of money at the beginning of every year.
- Our objective is to **maximise the amount of money remaining after the debt repayment to the PC**.

The Target

is to invest the minimal possible amount into the fund such that the probability that the debt can be fully repaid to the state stays above some pre-specified level.

The PC carries the investment risk where the state has the risk that the PC will not be able to repay the debt. It means in particular that we allow for negative values of the level b.

The Credibility Condition

We denote by $D_t(b)$ the part of the gains to be transferred to the debt account and by $R_t(b)$ the part remaining to the PC for the time horizon t.

Mathematically:

$$R_t(b) = e^{\mu t + \sigma W_t - \left(\max\limits_{0 \le s \le t}\{\mu s + \sigma W_s\} - \ln(1+b)\right)^+},$$

and

$$D_t(b) = (1+b)\left(\max_{0 \le s \le t}\{\mu s + \sigma W_s\} - \ln(1+b)\right)^+ .$$

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$$D_t(b) = (1+b)\left(\max_{0 \leq s \leq t}\{\mu s + \sigma W_s\} - \ln(1+b)\right)^+ .$$

We assume that the initial investment is $F_0 = \alpha(C_1 - C_0)$ with some positive α, and require for every period the **credibility condition**

$$\mathbb{P}\Big[F_0 D_t(b) \geq C_1 - C_0\Big] \geq p \Leftrightarrow \mathbb{P}\Big[D_t(b) \geq \frac{1}{\alpha}\Big] \geq p .$$

No animals were harmed in the making of this talk.

Optimising the Gain of the PC

Our target is to maximise $\mathbb{E}[R_1(b)]$ with respect to b by taking into account the credibility condition.

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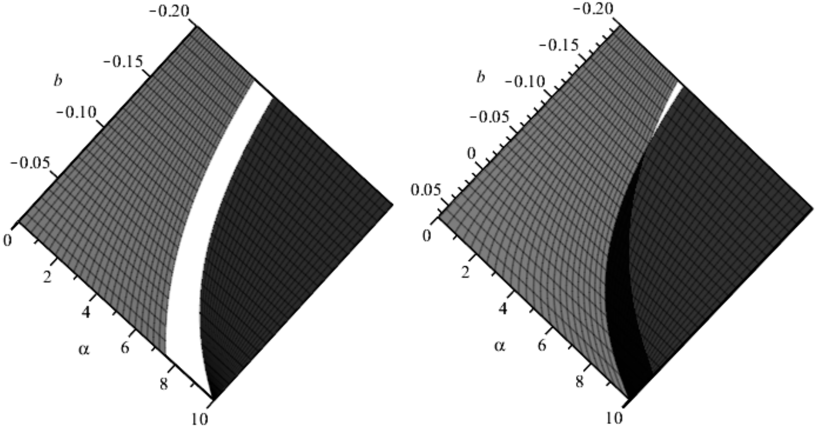
The following **profitability condition** checks if the payment of $C_1 - C_0$ into PAYG is more preferable:

$$\alpha(C_1 - C_0) - (C_1 - C_0) < \alpha(C_1 - C_0)\mathbb{E}[R_1(b)]$$

Intersections Between the Profitability and Credibility Conditions

Sets of (b, α) fulfilling the credibility (dark grey areas) and profitability (light grey areas) conditions for $p = 0.7$ (left) and $p = 0.5$ (right).



Comparison of the Repayment Procedures

The optimal repayment strategy for different values of α and time horizons t, $\mu = 0.04$, $\sigma = 0.2$.

t \ α	1	2	3	4	5	6	7	8	9	10
1	PAYG	PAYG	PAYG	C	C	C	C	C	C	C
2	PAYG	PAYG	C	C	C	C	C	C	C	C
4	PAYG	C	C	C	C	C	C	LS	LS	LS
6	PAYG	C	C	C	LS	LS	LS	LS	LS	LS
8	PAYG	C	C	LS	LS	LS	LS	LS	LS	LS
10	PAYG	C	LS	LS	LS	LS	LS	LS	LS	LS
20	LS	LS	LS	LS	LS	LS	LS	LS	LS	LS
40	LS	LS	LS	LS	LS	LS	LS	LS	LS	LS

Here, the probability of default for the state, i.e. the probability that the state will not get the full debt back, equals 50%.

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Thank You for Your Attention