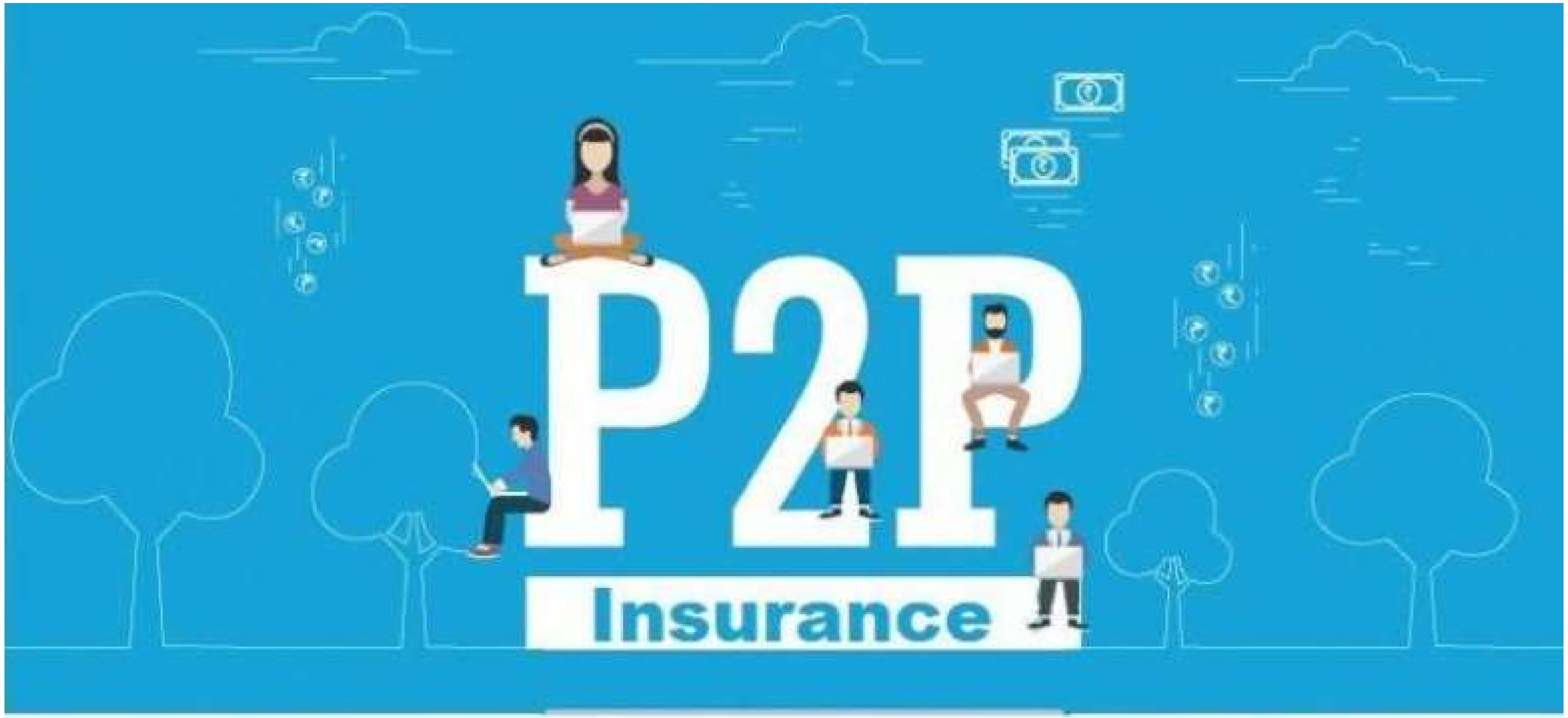

Actuarial models for P2P insurance

Actuarial modeling for P2P insurance

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In this talk, we present ideas from several set of papers.

- **Definition of the conditional mean risk sharing and its link with the size-biased transform**

Denuit, M., Dhaene, J. (2012). *Convex order and comonotonic conditional mean risk sharing*. Insurance : Mathematics and Economics 51, 265-270.

Denuit, M. (2019). *Size-biased transform and conditional mean risk sharing, with application to P2P insurance and tontines*. ASTIN Bulletin 49, 591-617.

Denuit, M. (2020a). *Size-biased risk measures of compound sums*. North American Actuarial Journal, in press.

- **Asymptotic behavior of the conditional mean risk sharing for large losses or large communities**

Denuit, M., Robert, C.Y. (2020a) *Large loss behavior of conditional mean risk sharing*. ASTIN Bulletin, in press.

Denuit, M., Robert, C.Y. (2020b). *From risk sharing to pure premium for a large number of heterogeneous losses*. Submitted for publication.

Denuit, M., Robert, C.Y. (2020c). *P2P insurance with reinsurance for a large number of participants*. Submitted for publication.

Denuit, M., Robert, C.Y. (2020d). *Efron's asymptotic monotonicity property in the Gaussian stable domain of attraction*. Submitted for publication.

▶ **The conditional mean risk sharing for dependent risks**

Denuit, M., Robert, C.Y. (2020e). *Conditional mean risk sharing for dependent risks using graphical models*. Submitted for publication.

Denuit, M., Robert, C.Y. (2020f). *Conditional tail expectation decomposition and conditional mean risk sharing for dependent and conditionally independent risks*. Submitted for publication.

▶ **P2P insurance models**

Denuit, M. (2020b). *Investing in your own and peers' risks : The simple analytics of P2P insurance*. European Actuarial Journal, in press.

Denuit, M., Robert, C.Y. (2020g). *From risk sharing to risk transfer : the analytics of collaborative insurance*. North American Actuarial Journal, in press.

Denuit, M., Robert, C.Y. (2020h). *Risk sharing under the dominant peer-to-peer insurance business models*. Submitted for publication.

1 Introduction

▶ **What is the sharing economy ?**

The sharing economy is an economic model defined as a peer-to-peer (P2P) based activity of acquiring, providing, or sharing access to goods and services that is often facilitated by a community-based on-line platform.

- The sharing economy has grown more and more in the past few years as technology has made connecting with others easier than ever. It often involves online platforms that connect buyers and sellers.

- The sharing economy is rapidly growing and evolving but faces significant challenges in the form of regulatory uncertainty and concerns about abuses.

▶ What about a long-standing and somewhat conservative industry like insurance ?

▶ **What is P2P insurance and how is it different from traditional insurance ?**

- The European Insurance and Occupational Pensions Authority (**EIOPA**) defines P2P insurance as

"*a risk-sharing digital network in which a group of individuals with mutual interests or similar risk profiles pool their "premiums" together to insure against a risk and to share the risk among them. Profits are commonly redistributed at the end of the year in case of good claims experience.*"

- The National Association of Insurance Commissioners (**NAIC**) defines P2P insurance as

"*an innovation that allows insureds to pool their capital, self-organise and self-administer their own insurance.*"

NAIC also outlines the core idea of P2P as a set of like-minded people with mutual interests grouping their insurance policies, introducing a sense of control, trust and transparency while at the same time reducing costs.

- **The cash-back mechanism**

  - In traditional insurance, reserved premiums not paid out in claims are typically held by insurers.

  - In P2P insurance, residual funds (excess premiums) from the paid premiums return to the group or to a charity when a smaller than anticipated number of claims are filed.

- **Other arguments pointed out by P2P insurance models (against regular insurance) are :**

  - save money through reduced costs,

  - reduce adverse selection,

  - increase transparency,

  - reduce inefficiencies,

  - reduce frauds,

- reduce conflicts between insurance carriers and their policyholders at the time of a claim.

- **There are three main types of P2P insurance models.**

  - the self-governing model,

  - the P2P broker model,

  - the P2P carrier model.

“**The self-governing model**” or the true P2P model

Such P2P organizations are made up of **self-governing user communities**.

- A communauty consists of **peers (teammates) that collectively manage all insurance functions**, such as : setting of insurance rules (i.e., what is covered, what documents need to be submitted, etc.), signing of new members, appraisal of claims and approval of reimbursements, payment of reimbursements.

- **Peers control via voting each decision** and are free to instantly delegate their votes to other peers, creating chains of trust.

- Communauties can be created on the basis of likeness of peers or insured objects : kind of insured object (e.g., car, house, health), kind of insured incident (e.g., collision, damage to 3rd party), social or professional affinity, home/work location (e.g., living in same town, working in same office building), etc.

The example of **TongJuBao**

- TongJuBao was launched in China in 2015.

- TongJuBao provides an example of P2P without any insurance company involved, showing the possible degree of disintermediation.

- Instead of partnering with or intermediating for insurers, TongJuBao is a matchmaker for people willing to join a common risk pool under an arrangement governed solely by civil law contracts.

- TongJuBao provides administration of the risk pools (using apps and chatbots to facilitate transparency and ease of interaction), but legal entitlement of pool members is only among themselves, not towards TongJuBao.

- TonJuBao provides an example of how the absence of insurance formalisms can result in new covers that respond to users'. E.g. TonJuBao has established mechanisms like discussion boards and voting procedures to implement new cover ideas (even when they are not endorsed by a majority).

"When People protect People"

P2P Protect Co. Ltd.
(TongJuBao.com)

“The P2P broker model”

Such a model is created/supported/managed by **an insurance broker**.

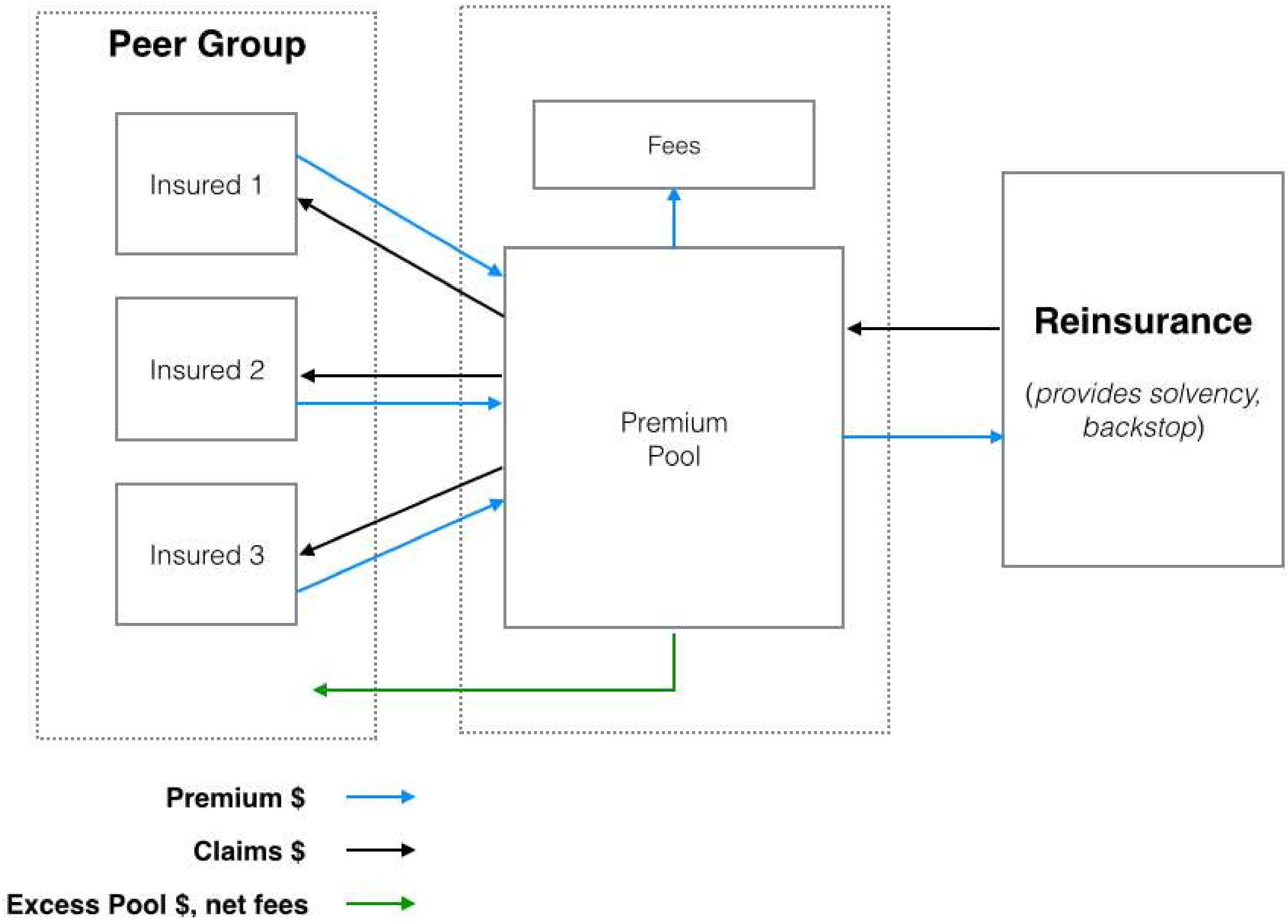
Its main distinctive features are :

- P2P insurance participants form groups (mainly online).

- A part of the insurance premiums paid flow into a group fund, **the other part to a third party (re)-insurance company.**

- If the pool is insufficient to pay for the claims of its members, the (re)-insurance carrier pays the excess from its retained premiums.

- Conversely, if the pool is "profitable" (i.e. has few claims), the "excess" is given back to the pool or to a cause the pool members care about.


Peer Group
Insured 1
Insured 2
Insured 3
Fees
Premium Pool
Reinsurance
(provides solvency, backstop)
Premium $
Claims $
Excess Pool $, net fees

The example of **Friendsurance**

- Friendsurance was launched in Germany in 2010.

- Friendsurance customers can select an insurance product offered by an insurance company through a partnership with Friendsurance and connect with other insureds who have a similar insurance need.

- These insureds are then placed in a pool and part of their premiums go to the insurer, another part goes to Friendsurance and the rest go in a cashback fund.

- Friendsurance indemnifies insureds for any small claims and the insurance company pays larger claims.

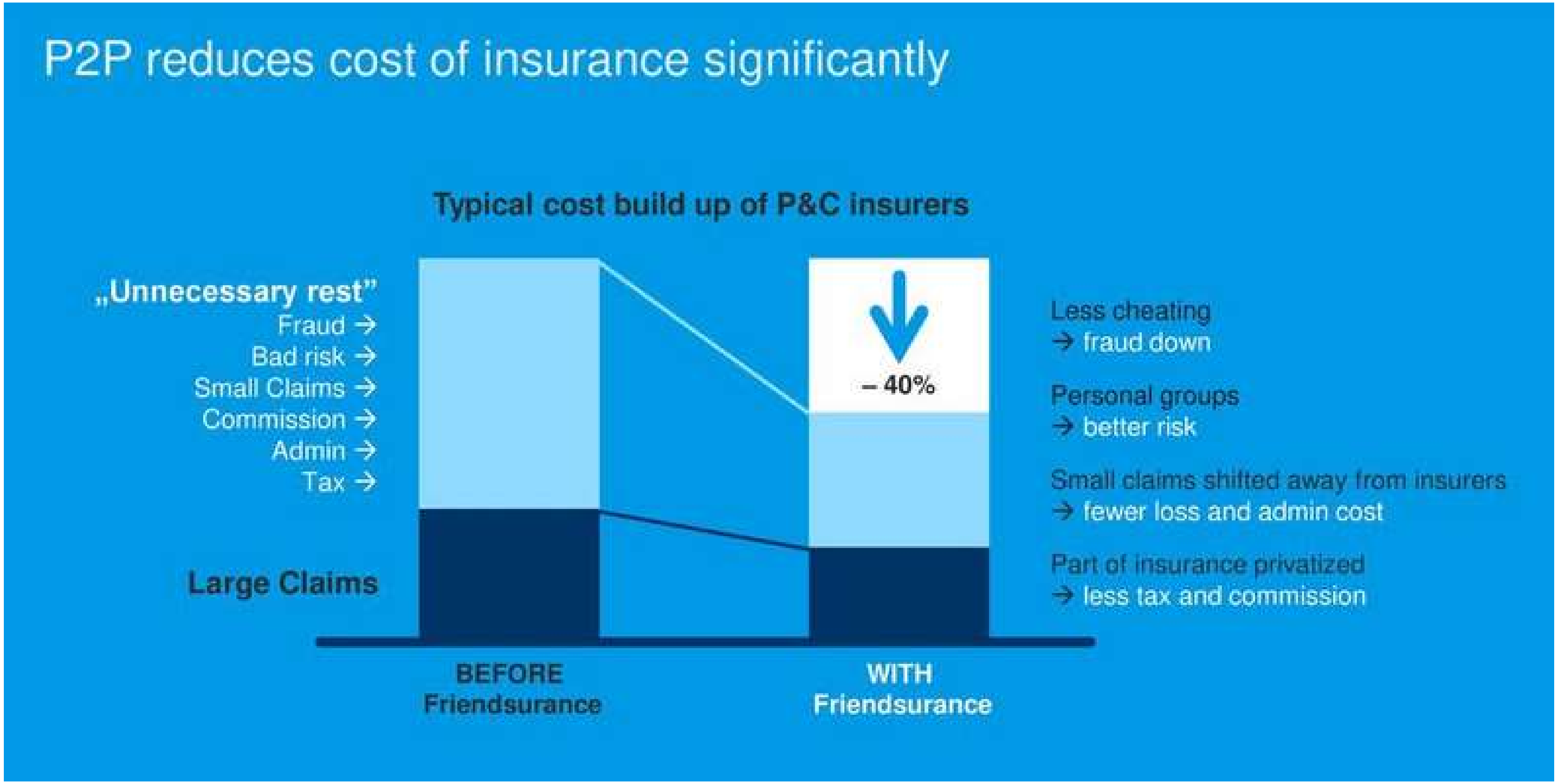
- As insureds submit larger claims covered by the main insurance company, the cashback fund decreases.

- At the end of the year, whatever money remaining in the fund is distributed back to insureds. Thus, customers are incentivized only to submit claims when truly necessary.

- By handling small claims directly, Friendsurance also decreases costs for the main insurance provider and consequently allows insureds to also pay lower premiums.

Friendsurance's slides

Friendsurance's slides



“The P2P carrier model”

- The carrier model easily fits in the established insurance sector, taking advantage of its risk-bearing capacity and expertise in claim settlement procedures.

- The **carrier model embeds the broker model into a regular insurance policy including a participation to the surplus**, but, in the same time, offering a fully digital and effective customer relationship.

- **Lemonade**, a licensed US insurance company founded in 2015, is the prototype example of this approach. Lemonade was the first U.S.-based P2P company to officially announce plans to operate as an insurance carrier.

Lemonade uses machine learning to go beyond satisfying customers and driving efficiencies to underwriting risks and managing claims.

Lemonade's business model is based on a transparent fee model, fast claim settlement, and social good. **It communicates its fee structure with its customers to achieve trustworthiness and transparency.**

Collected customer premium is utilized by Lemonade in the following manner :

- 20% flat fee for Lemonade
- 40% for reinsurance to cover major claims
- 40% for claims ; any surplus goes to charity

P2P insurance needs transparent and

well-established actuarial models to grow !

2 Risk sharing rules for an insurance pool

▶ We consider n participants to an insurance pool, numbered $i = 1, 2, \ldots, n$.

▶ Each of them faces a risk X_i. By risk, we mean a **non-negative random variable representing a monetary loss**.

- We assume that X_1, X_2, X_3, $\ldots$ are **mutually independent, but not necessarily identically distributed**.

- The **total risk** of the pool is denoted by $S = \sum_{i=1}^{n} X_i$.

▶ In a **risk pooling scheme**, each participant contributes **ex-post an amount** $h_i(s) \geq 0$ where $s = \sum_{i=1}^{n} x_i$ is the sum of the realizations $x_1, x_2, \ldots, x_n$ of $X_1, X_2, \ldots, X_n$ such that

$$\sum_{i=1}^{n} h_i(s) = s, \ s \geq 0.$$

▶ In the design of the scheme, it is important that the **sharing rule** represented by the functions h_i is **both intuitively acceptable and transparent**.

▶ A **fair risk sharing rule** is an allocation scheme such that the expected ex-post amount of a participant is equal to the expected value of his risk

$$\mathsf{E}[h_i(S)] = \mathsf{E}[X_i], \text{ for } i = 1, \ldots, n.$$

▶ To be successful, P2P insurance schemes should also require an appropriate risk sharing mechanism **recognizing the different distributions of the risks brought to the pool**.

$\Rightarrow$ Intuitively speaking, participants pooling “smaller” or “less variable” risks should contribute less to the realized total loss.

2.1 Proportional sharing rule

▶ In a **proportional sharing rule scheme**, participants agree to take a **fixed percentage of the total loss** S,

$$h_i^{\text{prop}}(s) = a_i s, \text{ for } i = 1, \ldots, n,$$

where $a_i \geq 0$ and $\sum_{i=1}^{n} a_i = 1$.

▶ If the proportional sharing rule is also imposed to be "**fair**", then

$$a_i = \frac{\mathsf{E}[X_i]}{\mathsf{E}[S]}.$$

▶ However such a proportional rule **does not take into the "variability"** of the risk X_i.

2.2 Conditional mean risk sharing rule

▶ The **conditional mean risk sharing rule** has been proposed by Denuit and Dhaene (2012). The ex-post contributions are defined as

$$h_i^*(S) = \mathsf{E}[X_i|S], \; i = 1, 2, \ldots, n.$$

Participant i must **contribute the expected value of the risk X_i brought to the pool, given the total loss S.**

▶ Clearly, the conditional mean risk sharing rule allocates the full risk S as we obviously have

$$\sum_{i=1}^{n} h_i^*(S) = \sum_{i=1}^{n} \mathsf{E}[X_i|S] = S$$

so that the sum of participants' contributions covers the entire loss S.

▶ The **conditional mean risk sharing** rule is **fair** since

$$\mathsf{E}[\mathsf{E}[X_i|S]] = \mathsf{E}[X_i].$$

▶ In the **expected utility setting, every risk-averse decision-maker prefers** $h_i^*(S)$ **over the initial risk** X_i so that the conditional mean risk sharing rule appears to be beneficial to all participants (as an application of Jensen's inequality).

▶ The conditional mean risk sharing rule does not require the prior elicitation of individual preferences beyond risk aversion.

▶ Denuit and Dhaene (2012) established that **the conditional mean risk sharing rule is Pareto-optimal for all risk-averse expected-utility maximizers, as long as every function** $s \mapsto \mathsf{E}[X_i|S=s]$ **is non-decreasing**

▶ Denuit and Robert (2020b) established that, under mild technical conditions, $\mathsf{E}[X_i|S]$ **tends to** $\mathsf{E}[X_i]$ **as the number** n **of participants increases**, with probability 1.

Conditional mean risk sharing and size-biased transform

Given **a non-negative random variable** X with distribution function F_X and strictly positive expected value $\mathsf{E}[X]$, a **size-biased version** of X, denoted by $\widetilde{X}$, is defined as

$$P[\widetilde{X} \leq t] = \frac{\mathsf{E}\Big[XI[X \leq t]\Big]}{\mathsf{E}[X]},$$

where $I[\cdot]$ denotes the indicator function (equal to 1 if the event appearing within the brackets is realized, and to 0 otherwise).

$\Rightarrow$ $\widetilde{X}$ is called the **size-biased transform** of X.

▶ If X has a density function f_X

$$f_{\widetilde{X}}(x) = \frac{xf_X(x)}{\mathsf{E}[X]}.$$

▶ Remark :

- **Compound sums** with absolutely continuous severities have **a probability mass at 0 and possess a probability density function over** $(0, \infty)$.

- Such a distribution is said to be **zero-augmented** and it appears to be relevant to examine the effect of size-biasing in this case.

- Assume that X is a zero-augmented risk, i.e. it is equal to 0 with probability $P[X = 0] > 0$ or strictly positive with probability $P[X > 0]$ and possesses the probability density function $f_{X|X>0}$ over $(0, \infty)$.

- Then, $\widetilde{X}$ is a strictly positive random variable with probability density function

$$f_{\widetilde{X}}(x) = \frac{x f_{X|X>0}(x)}{\mathrm{E}[X|X > 0]}.$$

Let us now give the reason why size-biasing appears to be useful in relation with the conditional mean risk sharing.

▶ Let $\widetilde{X}_1, \widetilde{X}_2, \ldots, \widetilde{X}_n$ be the corresponding size-biased versions of $X_1, X_2, \ldots, X_n$, assumed to be independent and independent of $X_1, X_2, \ldots, X_n$.

▶ It is proved in Denuit (2019) that, for any $s > 0$,

$$\mathsf{E}[X_i|S = s] = \frac{\mathsf{E}[X_i] f_{S-X_i+\widetilde{X}_i}(s)}{\sum_{j=1}^{n} \mathsf{E}[X_j] f_{S-X_j+\widetilde{X}_j}(s)} s.$$

Compound Binomial random variable

- The size-biased version of the compound Binomial random variable

$$X = \sum_{k=1}^{N} C_k$$

with N ~Binomial(ν, p) and claim severities C_k that are positive, absolutely continuous, independent and identically distributed as C, all these random variables being independent, is given by

$$\widetilde{X} =_d \sum_{k=1}^{N'} C_k + \tilde{C}$$

where N' ~Binomial$(\nu - 1, p)$, and where N', C_1, C_2, ..., $C_{\nu-1}$ and $\tilde{C}$ are mutually independent.

▶ Assume that the loss X_i brought by participant i to the insurance pool can be represented as

$$X_i = \sum_{k=1}^{N_i} C_{i,k} \text{ with } N_i \sim \text{Binomial}(\nu_i, p_i),\ i = 1, 2, \ldots,$$

where ν_i is a positive integer, $p_i \in (0,1)$, and where the claim severities $C_{i,k}$ are positive, absolutely continuous, distributed as C_i then

$$\mathsf{E}\left[X_i | S = s\right] = \frac{\mathsf{E}\left[X_i\right] f_{S - Y_{i,\nu_i} + \widetilde{C}_i}(s)}{\sum_{j=1}^{n} \mathsf{E}\left[X_j\right] f_{S - Y_{j,\nu_j} + \widetilde{C}_j}(s)} s$$

where $Y_{i,k} = I_{i,k} C_{i,k}$, $I_{i,k}$ being independent Bernoulli distributed random variables with common mean p_i and independent of $C_{i,k}$.

▶ Assume that $C_1, C_2, \ldots, C_n$ are identically distributed and $p_1 = ... = p_n$. Then

$$\mathsf{E}[X_i | S = s] = \frac{\nu_i}{\nu_\bullet} s$$

where $\nu_\bullet = \nu_1 + \ldots + \nu_n$.

Compound Poisson random variable

- The size-biased version of the compound Poisson random variable

$$X = \sum_{k=1}^{N} C_k$$

with $N \sim \text{Poisson}\,(\lambda)$ and claim severities C_k that are positive, absolutely continuous, independent and identically distributed as C, all these random variables being independent, is given by

$$\widetilde{X} =_d X + \tilde{C}$$

where X and $\tilde{C}$ are mutually independent.

▶ Assume that the loss X_i brought by participant i to the insurance pool can be represented as

$$X_i = \sum_{k=1}^{N_i} C_{i,k} \text{ with } N_i \sim \text{Poisson}(\lambda_i),\ i = 1, 2, \ldots,$$

where the claim severities C_{ik} are positive, absolutely continuous, distributed as C_i, all these random variables being independent, then

$$\mathsf{E}\left[X_i | S = s\right] = \frac{E\left[X_i\right] f_{S+\widetilde{C}_i}(s)}{\sum_{j=1}^{n} E\left[X_j\right] f_{S+\widetilde{C}_j}(s)} s.$$

▶ Assume that $C_1, C_2, \ldots, C_n$ are identically distributed. Then,

$$\mathsf{E}[X_i | S = s] = \frac{\lambda_i}{\lambda_\bullet} s$$

where $\lambda_\bullet = \lambda_1 + \ldots + \lambda_n$.

Compound Negative Binomial random variable

- The size-biased version of the compound Negative Binomial random variable

$$X = \sum_{k=1}^{N} C_k$$

with N $\sim$Negative Binomial(ξ, β) and claim severities C_k that are positive, absolutely continuous, independent and identically distributed as C, all these random variables being independent, is given by

$$\widetilde{X} =_d X + \tilde{C} + Z$$

where Z is a compound Negative Binomial sum $\sum_{k=1}^{M} C'_k$ with M $\sim$Negative Binomial$(1, \beta)$ and C'_k distributed as C_k, all these random variables being independent.

▶ Assume that the loss X_i brought by participant i to the insurance pool can be represented as

$$X_i = \sum_{k=1}^{N_i} C_{i,k} \text{ with } N_i \sim \text{Negative Binomial}(\xi_i, \beta_i),\ i = 1, 2, \ldots,$$

with N_i obeying the Negative Binomial(ξ_i, β_i) distribution with positive parameters β_i and ξ_i, and where the claim severities $C_{i,k}$ are positive, absolutely continuous, independent and distributed as C_i, all these random variables being independent. Then

$$\mathsf{E}\left[X_i | S = s\right] = \frac{\mathsf{E}\left[X_i\right] f_{S+\widetilde{C}_i+Z_i}(s)}{\sum_{j=1}^{n} \mathsf{E}\left[X_j\right] f_{S+\widetilde{C}_j+Z_j}(s)} s.$$

▶ Assume that $C_1, C_2, \ldots, C_n$ are identically distributed. If $\beta_1 = ... = \beta_n$ then

$$\mathsf{E}[X_i | S = s] = \frac{\xi_i}{\xi_\bullet} s$$

where $\xi_\bullet = \xi_1 + \ldots + \xi_n$.

▶ <u>Large-loss behavior of the conditional mean risk sharing :</u>

- Do the respective relative contributions of the n participants tend to stabilize when the total loss of the pool increases ?

- Equivalently, do there exist constants δ_i, $i = 1, 2, \ldots, n$, such that

$$\delta_i > 0 \text{ for all } i \text{ and } \sum_{i=1}^{n} \delta_i = 1$$

and

$$h_i^*(s) = \mathrm{E}[X_i | S = s] \sim \delta_i s \text{ for } i \in \{1, 2, \ldots, n\}.$$

▶ Assume that the loss X_i brought by participant i to the insurance pool can be represented as $X_i = \sum_{k=1}^{N_i} C_{i,k}$ where N_i is a counting random variable such that there exists $\varepsilon_i > 0$ with $\mathrm{E}\left[e^{\varepsilon_i N_i}\right] < \infty$, the claim severities $C_{i,k}$ are positive, absolutely continuous, distributed as C_i, all these random variables being independent.

- Moreover assume that the tail functions $\bar{F}_{C_i}$ satisfy

$$\bar{F}_{C_i}(x) \sim x^{-\alpha_i} L_i(x)$$

where $L_i(\cdot)$ are slowly varying functions and $\alpha_i > 1$ for $i = 1, ..., n$.

- The following results then hold true (Denuit and Robert (2020a)) :

  - If $\alpha_1 = ... = \alpha_n = \alpha$ and $L_i(x) \sim c_i L(x)$ with $c_i > 0$ for $i = 1, ..., n$, then

$$\mathsf{E}[X_i | S = s] \sim \frac{\mathsf{E}[N_i] c_i}{\sum_{j=1}^{n} \mathsf{E}[N_j] c_j} s \text{ for } i \in \{1, 2, \ldots, n\}.$$

  - If $\alpha_1 < \min\{\alpha_2, ..., \alpha_n\}$ then

$$\mathsf{E}[X_1 | S = s] \sim s \text{ and } \mathsf{E}\left[X_j | S = s\right] = o(s) \text{ for } j \in \{2, \ldots, n\}.$$

- <u>Large number of participants behavior of the conditional mean risk sharing :</u>

Denuit and Robert (2020b) study the asymptotic behavior of the conditional mean risk sharing as the number of participants tends to infinity.

- Under regularity conditons, then

$$\begin{aligned}\lim_{n\to\infty} h_i^{\textsf{prop}}(S) &= \mathsf{E}[X_i] \text{ with probability 1}\\ \lim_{n\to\infty} h_i^{\star}(S) &= \mathsf{E}[X_i] \text{ with probability 1.}\end{aligned}$$

Let $\mu_n = \sum_{i=1}^{n} \mathsf{E}[X_i]$, $s_n^2 = \sum_{i=1}^{n} \mathsf{V}[X_i]$.

- Under regularity conditons, then

$$\frac{\mu_n}{\mathsf{E}[X_i]s_n}\left(h_{1,n}^{\textsf{prop}}(S) - \mathsf{E}[X_i]\right) \xrightarrow{\mathcal{L}} \textsf{Normal}\,(0,1)\,,$$

$$\frac{s_n}{\sigma_i^2}\left(h_i^{\star}(S) - \mathsf{E}[X_i]\right) \xrightarrow{\mathcal{L}} \textsf{Normal}\,(0,1)\,.$$

3 A flexible carrier model

3.1 Entry price

▶ To ensure that the system does not run out of money, **participants must pay a provision, ex-ante,** so that the sum of the provisions is enough to cover the entirety of the losses, in all cases.

▶ We assume that premiums and provisions are computed according to the mean-value principle, that is, they are equal to the pure premium (or expected loss) increased by a proportional loading.

▶ The amount π_i paid by participant i who brings a loss X_i is thus supposed to be of the form

$$\pi_i = (1 + \theta_{\mathsf{P2P}})\mathsf{E}[X_i]$$

where θ_{P2P} is the loading applied within the P2P community.

▶ **Whatever the actual losses S, the system is designed in such a way that participants will never have to pay more than π_i and they may even be rewarded by a cash-back mechanism in case of favorable experience.**

▶ That is why **the upper layer of the total losses is ceded to a (re-)insurer** and that collaborative insurance is restricted to the lower layer.

▶ Denote as θ_{comm} the loading of a regular, commercial insurance product covering the same peril and as θ_{SL} the loading applied by the (re-)insurer covering the higher layer, exceeding the community's risk-bearing capacity.

▶ It is reasonable to have $\theta_{\text{SL}} < \theta_{\text{comm}}$ because of the reduced expenses faced when covering only the higher layer.

▶ We assume that the P2P community selects a loading θ_{P2P} comprised between θ_{SL} and θ_{comm}, that is,

$$\theta_{\text{SL}} < \theta_{\text{P2P}} \leq \theta_{\text{comm}}.$$

3.2 Determination of the retention capacity of the P2P community

- The amount w retained by the pool is the solution to the equation

$$\sum_{i=1}^{n} \pi_i = w + (1 + \theta_{\mathsf{SL}})\mathsf{E}[(S - w)_+].$$

In this way, the total income $\sum_{i=1}^{n} \pi_i$ allows the organizer to pay for losses up to w and to transfer the upper layer (w, ∞) to a (re-)insurer in exchange of the premium

$$\pi_{\mathsf{SL}} = (1 + \theta_{\mathsf{SL}})\mathsf{E}[(S - w)_+].$$

- The existence and unicity of the solution w are guaranteed under the assumed inequality $\theta_{\mathsf{SL}} < \theta_{\mathsf{P2P}}$.

- The cash-back mechanism is set up as soon as $S < w$.

3.3 Individual, ex-ante contributions

▶ Each participant is liable for an amount

$$w_i = F^{-1}_{\mathsf{E}[X_i|S]}\Big(F_S(w)\Big) = \mathsf{E}[X_i|S = w] \quad \text{with } \sum_{i=1}^{n} w_i = w,$$

such that

$$S \leq w \Leftrightarrow \mathsf{E}[X_i|S] \leq w_i \text{ for } i = 1, 2, \ldots, n.$$

▶ The contribution π_i can then be split into

$$\pi_i = \pi_i^{\mathsf{P2P}} + \pi_i^{\mathsf{SL}} \quad \text{where } \pi_i^{\mathsf{SL}} = (1 + \theta_{\mathsf{SL}})\mathsf{E}\Big[\Big(\mathsf{E}[X_i|S] - w_i\Big)_+\Big].$$

▶ The total loss S is well covered : the upper layer (w, ∞) is transferred to the (re-)insurer for the payment π_i^{SL} and

$$\sum_{i=1}^{n} \pi_i - \sum_{i=1}^{n} \pi_i^{\mathsf{SL}} = w \quad \text{with } \pi_{\mathsf{SL}} = \sum_{i=1}^{n} \pi_i^{\mathsf{SL}}.$$

- The lower layer $(0, w)$ is covered by the contributions

$$\begin{aligned}\pi_i^{\mathsf{P2P}} &= (1+\theta_{\mathsf{P2P}})\mathsf{E}\Big[X_i\Big] - (1+\theta_{\mathsf{SL}})\mathsf{E}\Big[\Big(\mathsf{E}[X_i|S] - w_i\Big)_+\Big] \\ &= (\theta_{\mathsf{P2P}} - \theta_{\mathsf{SL}})\mathsf{E}\Big[X_i\Big] + (1+\theta_{\mathsf{SL}})\bigg(\mathsf{E}\Big[X_i\Big] - \mathsf{E}\Big[\Big(\mathsf{E}[X_i|S] - w_i\Big)_+\Big]\bigg) \\ &= (\theta_{\mathsf{P2P}} - \theta_{\mathsf{SL}})\mathsf{E}\Big[X_i\Big] + (1+\theta_{\mathsf{SL}})\mathsf{E}\Big[\min\Big\{\mathsf{E}[X_i|S], w_i\Big\}\Big].\end{aligned}$$

- The cash-back mechanism generates the end-of-period cash-flows

$$\Big(w_i - \mathsf{E}[X_i|S]\Big)\mathsf{I}[S \leq w].$$

3.4 Two-level P2P insurance community

▶ Often, participants are partitioned into smaller communities corresponding to social groups (friends or relatives), or to individuals with common economic interest or located in the same geographic area, for instance.

▶ The system then proceeds in two steps :

- the total losses are first shared between the different groups,
- the part allocated to each group is then shared among its members.

▶ Different risk sharing mechanisms between the groups and among the members of each specific group can be chosen. We might e.g. :

- allocate the losses between groups with the help of the conditional mean risk sharing rule, in order to avoid any undue transfer,
- allocate the total losses within groups with the help of the simpler proportional mean risk allocation.

Future work :

▶ **Numerical implementation challenges** : even if large-pool approximations by individual expected losses are available, it remains necessary to be able to compute individual contributions to total losses for small or moderate-sized communities :

- *parametric approach* + *approximation formulas* : GLM + orthonormal polynomial series expansions when risks belong to the class of compound Panjer-Katz sums.

- *machine learning algorithms* to estimate a conditional expectation as a function of the sum of the losses and all the characteristics of the participants. Issues : the sum of losses is an endogeneous variable, the size of the vector of explanatory variables is very large.

▶ **Mitigating adverse-selection and moral-hazard**

- *a priori risk classification and team design*

- *a posteriori credibility corrections*

▶ **Opportunities for other types of insurance policies**