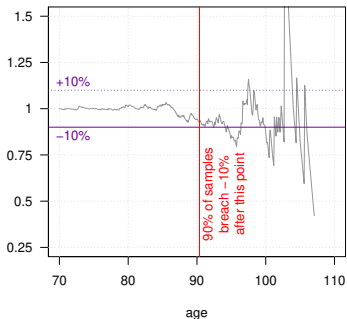


Wealth heterogeneity in a pooled annuity fund

Thomas Bernhardt, University of Manchester
joint work with Ge Qu

One World Actuarial Research Seminar
November 2024

Sample of experienced scaled income by age



Pooled annuity funds:

- give no guarantees to members
- adjust income according to the experienced mortality

Without new members, the pool cannot provide a stable income indefinitely:

- can look at the time until the income stays within thresholds with a high percentage note running inf/sup (not quantiles)

How do different initial savings influence the stability of the payments?

We will show that

In pooled annuity funds

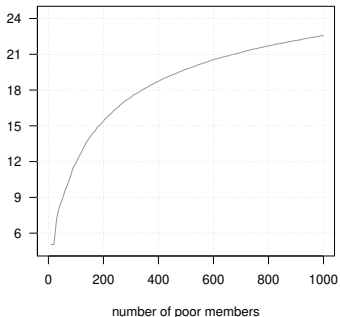
- wealth heterogeneity negatively affects the stability of income payments
- rich (high initial capital) members benefit from pooling their funds with poor (low initial capital)
- poor might be worse off in a larger heterogeneous pool than in a smaller homogeneous one
- we need to check whether a group benefits from pooling

We assume an i.i.d. cohort (only interested in wealth heterogeneity; fluctuations given by one process only)

We assume linear sharing rule (“by law”) and payments to survivors only

We assume a cohort of 1000 members, a mortality distribution, and fixed stability parameters (10% threshold, 90% certainty) to illustrate results

Stable income in years



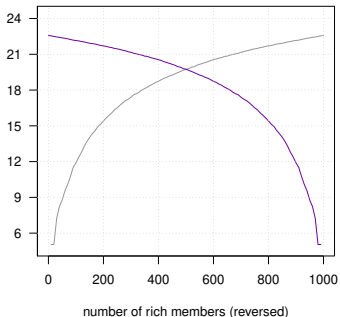
Consider two groups

- 1st group poor (low capital)
- 2nd group rich (high capital)

Consider a pool of 0 to 1000 poor members



Stable income in years

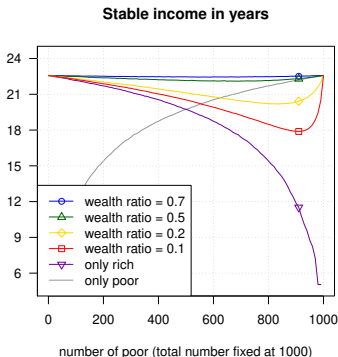


Consider two groups

- 1st group poor (low capital)
- 2nd group rich (high capital)

Consider a pool of 1000 to 0 rich members

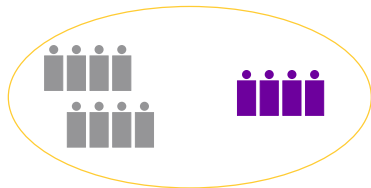


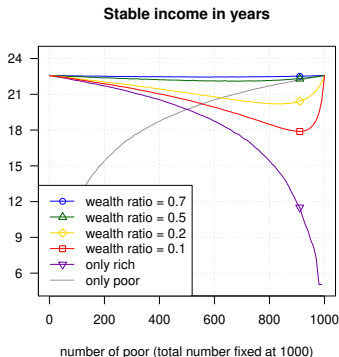


Consider two groups

- 1st group poor (low capital)
- 2nd group rich (high capital)

Mix poor and rich and change proportion and wealth inequality





Consider two groups

- wealth heterogeneity negatively affects the stability of income payments
- rich benefit from pooling their funds with poor
- poor might be worse off in a larger heterogeneous pool than in a smaller homogeneous one (yellow and red curve cross the grey curve)

Calculating income stability

Calculate the time t until unstable
(linear sharing, survivor only)

$$F(t) \approx \frac{1}{1 + \left(\frac{1-\varepsilon}{\varepsilon}\right)^2 \left(\Phi^{-1}\left(\frac{1-\beta}{2}\right)\right)^2 \frac{\sum_{i=1}^n s_i^2}{\left(\sum_{i=1}^n s_i\right)^2}}$$

- F mortality distribution fct.
- ε, β stability parameters
- Φ normal distribution function
- $(s_i)_{i=1}^n$ individual savings

our setting

Calculate the variance of the first
payment (requires i.i.d. cohort)

$$\text{Var}(C_j(1)) = \kappa_j \cdot {}_1p_x (1 - {}_1p_x) \frac{\sum_{i=1}^n s_i^2}{\left(\sum_{i=1}^n s_i\right)^2}$$

- $C_j(1)$ member j 's 1st payment
- κ_j investment related constant
- ${}_1p_x$ survival rate
- $(s_i)_{i=1}^n$ individual savings

actuarial fair annuity overlay fund
(by Donnelly, Guillén, Nielsen)

Can we tell whether the poor benefit from pooling with the rich?

- Yes, we can look at the “implied number”, IN for short, (larger means more stable)

$$IN = \frac{(\sum_{i=1}^n s_i)^2}{\sum_{i=1}^n s_i^2}$$

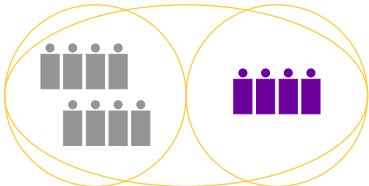
- The name comes from $IN \leq n$, i.e. bounded by the total number of members, (with equality when all members have the same savings)

Can we tell whether a group benefits from pooling their funds together?

- we need to maximise the implied number under all subgroups

Is the whole group the best subgroup = beneficial subgroup?

- we looked at three specific subgroups



- We need to look at 2^n subgroups!

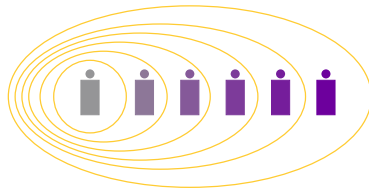
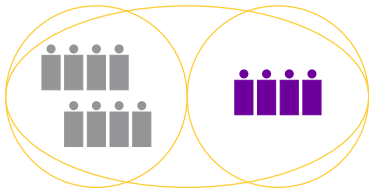


Can we tell whether a group benefits from pooling their funds together?

- we need to maximise the implied number under all subgroups

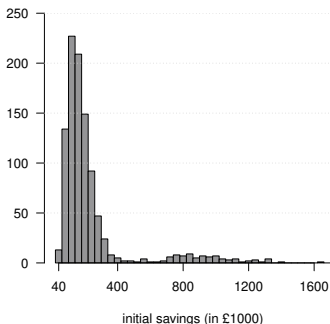
Is the whole group the best subgroup = beneficial subgroup?

- we looked at three specific subgroups
- look at groups with increasingly higher savings

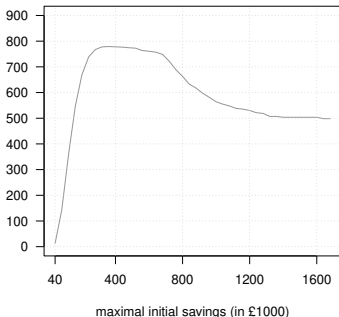


A company wants to start a pool

Number of retirees next year



Implied number



Suggests to introduce a contribution limit between £360k-680k

Whole stable for 19.7 years, beneficial one 21.6 (only 2 years more?)

- adding two years after 20 years is hard (“reaching end of life table”)
- not just adding 2 years but increase stability for first 20 years

Excluded retirees can contribute the contribution limit (again beneficial)

A company wants to start a pool

PENSIONERS POLICY INSTITUTE
PPI

The role of Collective Defined Contribution in decumulation



Modelling Appendix

Charges

Key assumptions

Mortality

Investment strategy and returns

Figure 4-8-1: Split table for new members

Pot value (£k (starting))	Members
£5,000	265
£20,000	206
£40,000	148
£75,000	138
£174,500	40
£624,500	40

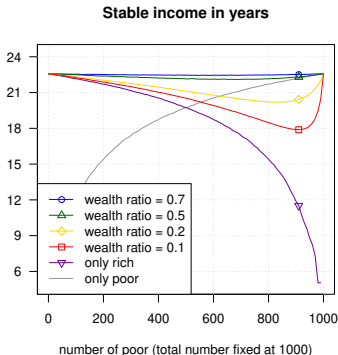
Pot value	Members
£5k	265
£20k	206
£40k	148
£75k	202
£174.5k	138
£624.5k	40

Funny example (but the papers point is about equity):

- £5k-group is as stable as the whole group of 999 members
- members with up to £75k is beneficial ($IN = 480$)

Setting up multi-employer pools

If the largest contribution to a pool is at most 2 times the smallest contribution, then the pool is guaranteed to be beneficial



countries have savings limits (for tax reasons), like £1000k

pension not advisable when savings too small

- Pool 1: £1000k-500k
- Pool 2: £500k-250k
- Pool 3: £250k-128k
- Pool 4: £128k-64k
- Pool 5: £64k-32k

individual pools are guaranteed to be beneficial no matter who joins

In pooled annuity funds

- wealth heterogeneity negatively affects the stability of income
- rich benefit from pooling their funds with poor
- poor might be worse off in a larger heterogeneous pool
- we need to check whether a group is beneficial
- we can use beneficial subgroups to decide on income brackets

Thank you very much!

Any questions or feedback? thomas.bernhardt@manchester.ac.uk